# **Two Way Repeated Measures ANOVA**

# **The Basic Idea**

The name of this test can be broken down to tell us the type of design with which it is used. The 'two-way' part of the name simply means that two independent variables have been manipulated in the experiment. The 'repeated measures' part of the name tells us that the same participants have been used in all conditions, and the ANOVA part tells us that we're comparing variances (i.e. we're looking at differences between means). Therefore, this design is used when you have two repeated-measures independent variables: each subject does all of the conditions in the experiment, and provides a score for each permutation of the two variables. As with any repeated-measures design, this has two advantages over independent-measures deigns:

- It is a very economical design, as it minimises the number of participants used;
- It should be very sensitive to the effects of our experimental manipulations because by using the same people throughout we control many potential confounding variables (such as motivation, intelligence etc.).

# **An Example**



Let's look at an example. At Christmas we normally leave treats for Santa Claus and his helpers (mince pies, a glass of sherry and a bucket of water for Rudolph). Santa Claus noticed that he was struggling to deliver all the presents on Christmas Eve and wondered whether these treats might be slowing down his Elves. So, one Christmas Santa did a little experiment. He randomly selected 10 Elves from his workforce and timed how long it took each of them to deliver the presents to 5 houses. Half of the elves were told that they could eat any mince

pies or Christmas pudding but that they must not have any sherry, while the other half were told to drink sherry but not to eat any food that was left for them. The following year Santa took the same 10 elves and again timed how long it took them to deliver presents to the same 5 houses as the previous year. This time, however, the ones who had drunk sherry the previous year were banned from drinking it and told instead to eat any mince pies or Christmas pudding. Conversely, the ones who had eaten treats the year before were told this year only to drink sherry and not to eat any treats. As such, over the two years each of the 10 elves was timed for their speed of present delivery after 1, 2, 3, 4 and 5 doses of sherry, and also after 1, 2, 3, 4, & 5 doses of mince pies.

### **Why do you think Santa got half of the elves to drink sherry the first year and ate treats the second year, while the other half ate treats the first year and drank sherry the second year?**

Think about the design of this study for a moment. We have the following variables:

- Independent Variable 1 is the treat that was consumed by the elves and it has 2 levels: Sherry or Mince Pies.
- Independent Variable 2 is the dose of the treat (remember each elf had a treat at the five houses to which they delivered and so the total quantity consumed increased across the houses). This variable has 5 levels: house 1, house 2, house 3, house 4 & house 5.
- The dependent variable was the time taken to deliver the presents to a given house (in milliseconds: elves deliver very quickly!)

These data could, therefore, be analysed with a  $2 \times 5$  two-way repeated-measures ANOVA. As with other ANOVA designs, there is in principle no limit to the number of conditions for each of



the independent variables in the experiment. In practice, however, you'll find that your participants get very bored and inattentive if there were too many conditions!

# *Partitioning Variance (ignore this if you're not interested)*

For this ANOVA, the variance will be partitioned in the following way:



Although this diagram looks very complex, the process is conceptually the same as that for twoway independent ANOVA (see your handout from a few weeks ago). That is, we are simply seeing how much of the total variability can be attributed to the independent variables that we have manipulated, how much can be explained by the interaction between the variables that we've manipulated, and how much is left over (the residual terms that cannot be explained by factors that we've controlled). So, we get terms representing the variables we've manipulated and errors (or residuals) associated with those terms.

The resulting ANOVA table will look like this (in general terms):



Like with the two-way independent ANOVA, we end up with three *F*-ratios: one for the first independent variable  $(F_A)$ , one for the second independent variable  $(F_B)$ , and one for the interaction of the two independent variables  $(F_{A \times B})$ . So in our example, we'd get an *F*-ratio for the effect of the type of treat  $(F_{\text{Teat}})$ , one for the effect of dose  $(F_{\text{Dose}})$ , and one for the interaction between the type of treat and the dose  $(F_{\text{Test} \times \text{Dose}})$ .

# *Degrees of Freedom (You can ignore this bit too if you want!)*

Although SPSS will do all the hard work calculating the sums of squares, mean squares, *F*-ratios and degrees of freedom, it is worth knowing where the degrees of freedom come from (especially as we have to report these values with each *F*-ratio). This will help you to understand the values that SPSS produces.

- **Total** *df*: As usual, the total *df* is the number of scores in the study minus one. If you look at the table of data, you'll see we have 100 data points (10 subjects each produced 10 scores, hence, 100 scores in total). Our *df* are therefore 99.
- **Between-Subjects** *df*: this is the number of participants minus 1. In this case we used 10 elves, so it is 9.
- *Df<sub>A</sub>***:** The degrees of freedom associated with the first independent variable are the number of levels of that variable (i.e. the number of experimental conditions for the first variable) minus 1. For the treat variable, we had two conditions, and so it would have 1 degree of freedom.
- *Df<sub>B</sub>*: The degrees of freedom associated with the second independent variable are the number of levels of that variable (i.e. the number of experimental conditions for the second variable) minus 1. For the dose variable, we had five conditions, and so it would have 4 degree of freedom.
- *Df*<sub>A x B</sub>: The degrees of freedom associated with the interaction effect are calculated in two stages.
- o First we work out how many 'cells' there are in the experimental design. This is simply the number of permutations of experimental conditions. So, in this example, for each treat there were 5 doses experienced by each elf. Therefore, mince pies had five doses, and sherry had five doses. So in total each participant took part in ten conditions. As such the number of cells in the experimental design is simply the total number of conditions. The degrees of freedom for the cells are this value minus one. So, here we had 10 cells and so we get 9 degrees of freedom.
- $\circ$  Second, we have to use the degrees of freedom from the cells to calculate the degrees of freedom for the interaction. Specifically, we use the degrees of freedom for the cells and subtract from it the degrees of freedom associated with the two independent variables.

 $df_{A\times B} = df_{cells} - df_A - df_B$ 

In this case we would get  $9 - 1 - 4 = 4$ .

- **Total Within Subjects** *df***:** To get the total degrees of freedom associated with the within-subjects components, we multiply the number of scores per subject minus 1, by the number of subjects. In this case, each elf took part in 10 conditions, and so produced 10 scores. The number of scores per subject minus one is, therefore, 9. We multiply this value by the number of participants (in this case 10 elves). We get  $9 \times 10 = 90$ .
- **Residual** *df***s:** To calculate all of the error *df*s, we multiply the between subjects *df* (in this case 9) by the *df* for the variable associated with the error. In this example, the first variable (treat) had 1 degree of freedom, and the between subjects *df* was 9, therefore, the error term for A has  $1 \times 9 = 9$  degrees of freedom. The second variable (dose) had 4 degrees of freedom, and the between subjects *df* was 9, therefore, the error term for B has  $4 \times 9 = 36$  degrees of freedom. Finally, the interaction term (treat  $\times$  dose) had 4 degrees of freedom, and the between subjects *df* was 9, therefore, the error term for A  $\times$  B has 4  $\times$  9 = 36 degrees of freedom.

# *Significance of the F-ratio (.. and this!)*

To calculate the significance of each *F*-ratio, you must use the degrees of freedom both for the effect and for its associated error term. So, to find the critical value for the effect of the treat variable we would use 1 and 9 degrees of freedom. Looking at a table for critical values of *F* (there's one on the handout for calculating one-way independent ANOVA by hand) you'll find the critical value is 5.12.

For the dose variable and the treat  $\times$  dose interaction, we use 4 and 36 degrees of freedom and find a critical value of approximately 2.69. **Luckily for us SPSS calculates the exact probability of obtaining a given** *F-***value by chance. If this value is less than 0.05 we know that the effect is significant by the criterion that psychologists use.**

# **Doing Factorial Repeated Measures ANOVA on SPSS (Don't ignore this!)**

# *The Main Analysis*

To enter these data into SPSS we need to recap the golden rule of the data editor, which states that each row represents a single participant's data. If a person participates in all experimental conditions (in this case all elves experience both Sherry and Mince Pies in the different doses) then each experimental condition must be represented by a column in the data editor. In this experiment there are ten experimental conditions and so the data need to be entered in ten columns (so, the format is identical to the original table in which I put the data). You should create the following ten variables in the data editor with the names as given. For each one, you should also enter a full variable name for clarity in the output.



Once these variables have been created, enter the data as in Table earlier in this handout. If you have problems entering the data then use the file **santa.sav** from the course website. To access the *define factors* dialog box use the menu path **Analyze**⇒**General Linear Model**⇒**Repeated Measures …**. In the *define factors* dialog box you are asked to supply a name for the within-subject (repeated measures) variable. In this case there are two within-subject factors: **treat** (Sherry or Mince Pie) and **dose** (1, 2, 3 4 or 5 doses). Replace the word *factor1* with the word *treat*. When you have given this repeated measures factor a name, you have to tell the computer how many levels there were to that variable. In this case, there were two types of treat, so we have to enter the number 2 into the box labelled *Number of Levels*. Click on  $\mathbb{R}^d$  to add this variable to the list of repeated measures variables. This variable will now appear in the white box at the bottom of the dialog box and appears as *treat(2)*. We now have to repeat this process for the second independent variable. Enter the word *dose* into the space labelled *Within-Subject Factor Name* and then, because there were five levels of this variable, enter the number 5 into the space labelled *Number of Levels*. Click on **Addam** to include this variable in the list of factors; it will appear as *dose(5)*. The finished dialog box is shown in Figure 1. When you have entered both of the within-subject factors click on  $\mathbb{P}^{\text{eff}}$  to go to the main dialog box.



**Figure 1:** *Define factors* dialog box for factorial repeated measures ANOVA

The main dialog box is essentially the same as when there is only one independent variable (see previous handout) except that there are now ten question marks (Figure 2). At the top of the *Within-Subjects Variables* box, SPSS states that there are two factors: **treat** and **dose**. In the box below there is a series of question marks followed by bracketed numbers. The numbers in brackets represent the levels of the factors (independent variables).



In this example, there are two independent variables and so there are two numbers in the brackets. The first number refers to levels of the first factor listed above the box (in this case **treat**). The second number in the bracket refers to levels of the second factor listed above the box (in this case **dose**). As with one-way repeated measures ANOVA, you are required to replace these question marks with variables from the list on the left-hand side of the dialog box. With between-group designs, in which coding variables are used, the levels of a particular factor are specified by the codes assigned to them in the data editor. However, in repeated measures designs, no such coding scheme is used and so we determine which condition to assign to a level at this stage. For example, if we entered **sherry1** into the list first, then SPSS will treat sherry as the first level of **treat**, and dose 1 as the first level of the **dose** variable. However, if we entered **pie5** into the list first, SPSS would consider mince pies as the first level of **treat**, and dose 5 as the first level of **dose**



**Figure 2** 

It should be reasonably obvious that it doesn't really matter which way round we specify the treats, but is very important that we specify the doses in the correct order. Therefore, the variables could be entered as follows:



When these variables have been transferred, the dialog box should look exactly like Figure 3. The buttons at the bottom of the screen have already been described for the one independent variable case and so I will describe only the most relevant.



**Figure 3** 

# *Graphing Interactions*

When we had only one independent variable, we ignored the *plots* dialog box; however, if there are two or more factors, the *plots* dialog box is a convenient way to plot the means for each level of the factors. To access this dialog box click on **Fides.** Select **dose** from the variables list on the left-hand side of the dialog box and transfer it to the space labelled *Horizontal Axis* by clicking on . In the space labelled *Separate Lines* we need to place the remaining independent variable: **treat**. As before, it is down to your discretion which way round the graph is plotted, but it actually makes sense this time to have dose on the horizontal axis. When you have moved the two independent variables to the appropriate box, click on  $\mathbb{R}^d$  and this interaction graph will be added to the list at the bottom of the box (see Figure 4). When you have finished specifying graphs, click on **Continue** to return to the main dialog box.



**Figure 4** 

### *Other Options*

You should notice that *post hoc* tests are disabled for solely repeated measures designs. Therefore, the only remaining options are in the *options* dialog box, which is accessed by clicking on . The options here are the same as for the one-way ANOVA. I recommend selecting some descriptive statistics and you might also want to select some multiple comparisons by selecting all factors in the box labelled *Factor(s) and Factor Interactions* and transferring them to the box labelled *Display Means for* by clicking on **I** (see Figure 5). Having selected these variables, you should tick the box labelled *Compare main effects* ( $\sqrt{P}$  Compare main effects) and select an appropriate correction (I chose Bonferroni).



**Figure 5** 

# **Interpreting the Output from Factorial Repeated Measures ANOVA**

#### *Descriptives and Main Analysis*



SPSS Output 1 shows the initial output from this ANOVA. The first table merely lists the variables that have been included from the data editor and the level of each independent variable that they represent. This table is more important than it might seem, because it enables you to verify that the variables in the SPSS data editor represent the correct levels of the

independent variables. The second table is a table of descriptives and provides the mean and standard deviation for each of the nine conditions. The names in this table are the names I gave the variables in the data editor (therefore, if you didn't give these variables full names, this table will look slightly different).

The descriptives are interesting in that they tell us that the variability among scores was greatest after 5 Sherries and was generally higher when sherry was consumed (compare the standard deviations of the levels of the sherry variable compared to those of the mince pie variable). The values in this table will help us later to interpret the main effects of the analysis.



#### **SPSS Output 1**

SPSS Output 2 shows the results of Mauchly's sphericity test (see section 9.1.3 of Field, 2000) for each of the three effects in the model (two main effects and one interaction). The significance values of these tests indicate that the main effect of **dose** has violated this assumption and so the *F*-value should be corrected (see section 9.2.4.2. of Field, 2000). For the main effect of **treat** and the interaction the assumption of sphericity is met (because *p* > 0.05) so we need not correct the *F*-ratios for these effect.



**Mauchly's Test of Sphericityb**

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

b.

Design: Intercept Within Subjects Design: TREAT+DOSE+TREAT\*DOSE

**SPSS Output 2** 

SPSS Output 3 shows the results of the ANOVA (with corrected *F* values). The output is split into sections that refer to each of the effects in the model and the error terms associated with these effects (a bit like the general table earlier on in this handout). The interesting part is the significance values of the *F*-ratios. If these values are less than 0.05 then we can say that an effect is significant. Looking at the significance values in the table it is clear that there is a significant effect of the type of treat consumed by the elves, a significant main effect of the number of treats consumed (dose), and a significant interaction between these two variables. I will examine each of these effects in turn. As a final note compare the degrees of freedom to those we generated earlier on!



#### **Tests of Within-Subjects Effects**

#### **SPSS Output 3**

# *The Effect of Treat*

The first part of SPSS Output 3 tells us the effect of the type of treat consumed by the elves. For this effect there was no violation of sphericity and so we can look at the uncorrected *F*-ratios. Therefore, we should report that there was a significant main effect of the type of treat (*F*(1, 9) = 34.08, *p* < 0.001). This effect tells us that if we ignore the number of treats consumed, the elves were slower at delivering presents after one type of treat than after the other type.



You can request that SPSS produce means of the main effects (see Field, 2000 section 9.3.4) and if you do this, you'll find the table in SPSS Output 4 in a section headed *Estimated Marginal Means*.<sup>1</sup> SPSS Output 4 is a table of means for the main effect of treat with the associated standard errors. The levels of this

variable are labelled 1 and 2 and so we must think back to how we entered the variable to see which row of the table relates to which condition. We entered this variable with  $\overline{a}$ 



<sup>1</sup> These means are obtained by taking the average of the means in SPSS Output 1 for a given condition. For example, the mean for the mince pie condition (ignoring the dose) is

$$
\overline{X}_{\text{mincepie}} = \left(\overline{X}_{\text{1mincepie}} + \overline{X}_{\text{2mincepies}} + \overline{X}_{\text{3mincepies}} + \overline{X}_{\text{4mincepies}} + \overline{X}_{\text{5mincepies}}\right) / 5
$$
\n
$$
= (9.3 + 13.4 + 17.0 + 20.1 + 26.8) / 5 = 17.32.
$$

the sherry condition first and the mince pie condition last. Figure 1 uses this information to display the means for each condition. It is clear from this graph that mean delivery times were higher after sherry (mean = 25.64) than after mince pies (mean = 17.32). Therefore, sherry slowed down present delivery significantly compared to mince pies.



# **SPSS Output 4** Figure 1



### *The Effect of Dose*

SPSS Output 3 also tells us the effect of the number of treats consumed (dose) by the elves. For this effect there was a violation of sphericity and so we must look at the corrected *F*-ratios. All of the corrected values are highly significant and so we can report the Greenhouse-Geisser corrected values as these are the most conservative. We should report that there was a significant main effect of the number of treats consumed  $(F(2.21, 19.88) = 83.49, p < 0.001)$ . Note the degrees of freedom represent the Greenhouse-Geisser corrected values.

This effect tells us that if we ignore the type of treats that was consumed, the elves were slower at delivering presents certain numbers of treats. We don't know from this effect, which doses in particular slowed the elves down, but we could look at this with some kind of contrast (like a repeated contrast — see Field, 2000, Table 7.6, p. 272).





### SPSS Output 5 Figure 2

If we requested means of the main effects (see Field, 2000 section 9.3.4) then you'll see the table in SPSS Output 5, which is a table of means for the main effect of dose with the associated standard errors. The levels of this variable are labelled 1, 2, 3, 4,  $\&$  5 and so we must think back

to how we entered the variables to see which row of the table relates to which condition. Figure 2 uses this information to display the means for each condition. It is clear from this graph that mean delivery times got progressively higher as more treats were consumed (in fact the trend looks linear).

### *The Interaction Effect (Treat* × *Dose)*

SPSS Output 3 indicated that the number of treats consumed interacted in some way with the type of treat. The means for all conditions can be seen in SPSS Output 6 (and these values are the same as in the table of descriptives). The interaction did not violate sphericity and so we can report from the ANOVA table that there was a significant interaction between the type of treat consumed and the number of treats consumed  $(F(4, 36) = 20.73, p < 0.001)$ . This effect tells us that the effect of consuming more treats was stronger for one of the treats than for the other.









We can use the means in SPSS Output 6 to plot an interaction graph, which is essential for interpreting the interaction. Figure 3 shows two interaction graphs of these data (just to illustrate that you can present them as both bars or lines). The graph shows that the pattern of

responding for the two treats is very similar for small doses (the lines are almost identical for 1 and two doses and the bars are the same height). However, as more treats are consumed, the effect of drinking sherry becomes more pronounced (delivery times are higher) than when mince pies are eaten. This effect is shown by the fact that the line representing sherry starts to deviate away from the line for mince pies (the lines become non-parallel). In the bar chart this is shown by the



increasingly large differences between the pairs of bars for large numbers of treats. To verify the interpretation of the interaction effect, we would need to look at some contrasts (see Field, 2000, chapter 9). However, in general terms, Santa Claus should conclude that the number of treats consumed had a much greater effect in slowing down elves when the treat was sherry (presumably because they all get shit-faced and start staggering around being stupid), but much less of an effect when the treats were mince pies although even the pies did slow them down to some extent).

*This handout doesn't particularly contain material from:* 

**Field, A. P. (2000).** *Discovering statistics using SPSS for Windows: advanced techniques for the beginner***. London: Sage.**

*but please consult this book (chapter 9) for details on how to use SPSS to analyse factorial ANOVAs with repeated measures.*

# **Example 2:**

A clinical psychologist was interested in the effects of antidepressants and cognitive behaviour therapy on suicidal thoughts. Four depressives took part in four conditions: placebo tablet with no therapy for one month, placebo tablet with cognitive behaviour therapy (CBT) for one month, antidepressant with no therapy for one month, and antidepressant with cognitive behaviour therapy (CBT) for one month. The order of conditions was fully counterbalanced across the 4 participants. Participants recorded the number of suicidal thoughts they had during the final week of each month. The data are below:



The SPSS output for these data is as follows:

#### **Within-Subjects Factors** Measure: MEASURE\_1



#### **Descriptive Statistics**



**Mauchly's Test of Sphericityb**



proportional to an identity matrix.

a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are<br>displayed in the Tests of Within-Subjects Effects table. b.

Design: Intercept Within Subjects Design: DRUG+THERAPY+DRUG\*THERAPY

#### **Tests of Within-Subjects Effects**



**1. DRUG**



#### **2. THERAPY**







- Interpret the results of this analysis.
- Try carrying out the Analysis on SPSS.
- Work through Field (2000) Chapter 9.

# **Example 3:**

Measure: MEASURE\_1

DRUG 1 2

THERAPY 1 2 1  $\mathcal{L}$ 

In a previous handout we came across the beer-goggles effect: a severe perceptual distortion after imbibing vast quantities of alcohol. The specific visual distortion is that previously unattractive people suddenly become the hottest thing since Spicy Gonzalez' extra hot Tabascomarinated chilies. In short, one minute you're standing in a zoo admiring the Orangutans, and the next you're wondering why someone would put Gail Porter (or whatever her surname is now) into a cage. Anyway, in that handout, a blatantly fabricated data set demonstrated that the beer-goggles effect was much stronger for men than women, and took effect only after two pints. Imagine we wanted to follow this finding up to look at what factors mediate the beer goggles effect. Specifically, we thought that the beer goggles effect might be made worse by the fact that it usually occurs in clubs, which have dim lighting. We took a sample of 26 men (because the effect is stronger in men) and gave them various doses of alcohol over four different weeks (0 pints, 2 pints, 4 pints and 6 pints of lager). This is our first independent variable, which we'll call *alcohol consumption*, and it has four levels. Each week (and, therefore, in each state of drunkenness) participants were asked to select a mate in a normal club (that had dim lighting) and then select a second mate in a specially designed club that had bright lighting. As such, the second independent variable was whether the club had dim or bright lighting. The outcome measure was the attractiveness of each mate as assessed by a panel of independent judges. To recap, all participants took part in all levels of the alcohol consumption variable, and selected mates in both brightly- and dimly-lit clubs. The data are in the file **BeerGogglesLighting.sav** on the course website, analyse them with a two-way repeatedmeasures ANOVA.