Overview of Lecture

- Factorial Designs
- Experimental Design Names
- Partitioning the Variability
- The Two-Way Between Groups ANOVA
- Evaluating the Null Hypotheses
  - Main effects
  - Interactions
  - Analytical Comparisons

Factorial Design

- Much experimental psychology asks the question:
  - What effect does a single independent variable have on a single dependent variable?
- It is quite reasonable to ask the following question as well.
  - What effects do multiple independent variables have on a single dependent variable?
- Designs which include multiple independent variables are known as factorial designs.

An example factorial design

- If we were looking at GENDER and TIME OF EXAM, these would be two independent factors
  - GENDER would only have two levels: male or female
  - TIME OF EXAM might have multiple levels, e.g.
    - morning, noon or night
  - This is a factorial design

Experimental Design Names

- The name of an experimental design depends on three pieces of information
  - The number of independent variables
  - The number of levels of each independent variable
  - The kind of independent variable
    - Between Groups
    - Within Subjects (or Repeated Measures)

Experimental design names

- If there is only one independent variable then
  - The design is a one-way design (e.g. does coffee drinking influence exam scores)
- If there are two independent variables
  - The design is a two-way design (e.g. does time of day or coffee drinking influence exam scores).
- If there are three independent variables
  - The design is a three-way design (e.g. does time of day, coffee drinking or age influence exam scores).
Experimental Design Names

- If all the IVs are between groups then
  - It is a Between Groups design
- If all the IVs are repeated measures
  - It is a Repeated Measures design
- If at least one IV is between groups and at least one IV is repeated measures
  - It is a Mixed or Split-Plot design

Experimental design names

- Three IVs
  - IV 1 is between groups and has two levels (e.g. a.m., p.m.)
  - IV 2 is between groups and has two levels (e.g. coffee, water).
  - IV 3 is repeated measures and has 3 levels (e.g. 1st year, 2nd year and 3rd year).
- The design is:
  - A three-way (2x2x3) mixed design.

Analysis of a 2-way between groups design using ANOVA

- To analyse the two-way between groups design we have to follow the same steps as the one-way between groups design
  - State the Null Hypotheses
  - Partition the Variability
  - Calculate the Mean Squares
  - Calculate the F-Ratios

Null Hypotheses

- There are 3 null hypotheses for the two-way (between groups design).
  - The means of the different levels of the first IV will be the same, e.g. \( \mu_{1a} = \mu_{2a} = \mu_{3a} \)
  - The means of the different levels of the second IV will be the same, e.g. \( \mu_{1b} = \mu_{2b} = \mu_{3b} \)
  - The differences between the means of the different levels of the interaction are not the same, e.g. \( \mu_{1ab} = \mu_{2ab} = \mu_{3ab} \)

An example null hypothesis for an interaction

- The differences between the levels of factor A are not the same.
  \( \bar{Y}_{a1} - \bar{Y}_{a2} = \bar{Y}_{a3} - \bar{Y}_{a4} \)
Partitioning the variability

• If we consider the different levels of a one-way ANOVA then we can look at the deviations due to the between groups variability and the within groups variability:

\[ \text{SS}_{\text{B}} = \sum \text{(dev. between groups)}^2 = \sum \bar{y}_{.j}^2 \bar{y}_{.i}^2 \]

• If we substitute AB into the above equation we get

\[ \text{SS}_{\text{AB}} = \sum (\text{dev. between groups})^2 = \sum \bar{y}_{ij}^2 \bar{y}_{ij}^2 \]

• This provides the deviations associated with between and within groups variability for the two-way between groups design.

The mean squares

• In order to calculate F-Ratios we must calculate an Mean Square associated with
  - The Main Effect of the first IV
  - The Main Effect of the second IV
  - The Interaction
  - The Error Term

The sum of squares

• The sums of squares associated with the two-way between groups design follows the same form as the one-way

\[ \text{SS}_\text{O} = \text{SS}_\text{A} + \text{SS}_\text{B} + \text{SS}_\text{AB} + \text{SS}_\text{Res} \]

• We need to calculate a sum of squares associated with the main effect of A, a sum of squares associated with the main effect of B, a sum of squares associated with the effect of the interaction.

• From these we can estimate the variability due to the two variables and the interaction and an independent estimate of the variability due to the error.

The mean squares

• The main effect mean squares are given by:

\[ \text{MS}_\text{A} = \frac{\text{SS}_\text{A}}{d\text{f}_\text{A}} \text{ where } d\text{f}_\text{A} = a \cdot (a-1) \]

\[ \text{MS}_\text{B} = \frac{\text{SS}_\text{B}}{d\text{f}_\text{B}} \text{ where } d\text{f}_\text{B} = b \cdot (b-1) \]

• The interaction mean squares is given by:

\[ \text{MS}_\text{AB} = \frac{\text{SS}_\text{AB}}{d\text{f}_\text{AB}} \text{ where } d\text{f}_\text{AB} = (a-1) \cdot (b-1) \]

• The error mean square is given by:

\[ \text{MS}_\text{Res} = \frac{\text{SS}_\text{Res}}{d\text{f}_\text{Res}} \text{ where } d\text{f}_\text{Res} = ab \cdot c \cdot (c-1) \]

The between groups deviation can be thought of as a deviation that is comprised of three effects.

\[ \text{SS}_\text{B} = \text{SS}_\text{A} + \text{SS}_\text{B} + \text{SS}_\text{AB} \]

In other words the between groups variability is due to the effect of the first independent variable A, the effect of the second variable B, and the interaction between the two variables AB.
The F-ratios

- The F-ratio for the first main effect is:
  \[ \frac{M_{A}}{M_{\text{error}}} \]
- The F-ratio for the second main effect is:
  \[ \frac{M_{B}}{M_{\text{error}}} \]
- The F-ratio for the interaction is:
  \[ \frac{M_{AB}}{M_{\text{error}}} \]

Results of ANOVA

- When an analysis of variance is conducted on the data (using Experstat) the following results are obtained

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Squares</th>
<th>F</th>
<th>p</th>
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<tbody>
<tr>
<td>A (Lectures)</td>
<td>432.450</td>
<td>1</td>
<td>432.450</td>
<td>37.004</td>
<td>0.000</td>
</tr>
<tr>
<td>B (Worksheets)</td>
<td>0.450</td>
<td>1</td>
<td>0.450</td>
<td>0.036</td>
<td>0.846</td>
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<td>Error</td>
<td>184.000</td>
<td>18</td>
<td>11.500</td>
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<td></td>
</tr>
</tbody>
</table>

What does it mean? - Main effects

- A significant main effect of Factor A (lectures)
  - "There was a significant main effect of lectures \((F_{1,16}=37.604, \text{MSe}=11.500, p<0.001}\). The students who attended lectures on average scored higher \((\text{mean}=22.100)\) than those who did not \((\text{mean}=12.800)\).
- No significant main effect of Factor B (worksheets)
  - "The main effect of worksheets was not significant \((F_{1,16}=0.039, \text{MSe}=11.500, p=0.846)\)"

What does it mean? - Interaction

- A significant interaction effect
  - "There was a significant interaction between the lecture and worksheet factors \((F_{1,16}=18.178, \text{MSe}=11.500, p<0.001})" (mean=22.100)
  - However, we cannot at this point say anything specific about the differences between the means unless we look at the null hypothesis

\[ \mu_{\text{yes, lectures}, \text{yes, worksheets}} = \mu_{\text{yes, lectures}, \text{no, worksheets}} = \mu_{\text{no, lectures}, \text{yes, worksheets}} = \mu_{\text{no, lectures}, \text{no, worksheets}} \]

<table>
<thead>
<tr>
<th>LECTURES WORKSHEETS</th>
<th>Mean</th>
<th>Std Error</th>
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<tr>
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<td>yes</td>
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<td>no</td>
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Simple main effects analysis

- We can think of a two-way between groups analysis of variance as a combination of smaller one-way analyses.
- The analysis of simple main effects partitions the overall experiment in this way

\[ M_{\text{Lectures}} = \frac{1}{2} \left( M_{\text{yes, lectures}, \text{yes, worksheets}} + M_{\text{no, lectures}, \text{yes, worksheets}} \right) \]

\[ M_{\text{Worksheets}} = \frac{1}{2} \left( M_{\text{yes, lectures}, \text{yes, worksheets}} + M_{\text{no, lectures}, \text{yes, worksheets}} \right) \]
Results of a simple main effects analysis

- Using ExperStat it possible to conduct a simple main effects analysis relatively easily

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
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<tr>
<td>Error Term</td>
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<td>16</td>
<td>11.500</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

What does it mean? - Simple main effects of Lectures

- No significant simple main effect of lectures at worksheets (yes)
  - "There was no significant difference between those students who did attend lectures (mean=19.20) or did not attend lectures (mean=16.00) when they completed worksheets (F<sub>1,16</sub>=2.228, MSe=11.500, p=0.155)."
- Significant simple main effect of lectures at worksheets (no)
  - "There was a significant difference between those students who did attend lectures (mean=25.00) or did not attend lectures (mean=19.80) when they did not complete worksheets (F<sub>1,16</sub>=81.557, MSe=11.500, p<0.001). When students who attended lectures did not complete worksheets they scored higher on the exam than those students who neither attended lectures nor completed worksheets."